



An LTL Model Checker for Discrete Time Markov Chains

UIUC

YoungMin Kwon



Contents

- What can be checked
- Model and Specification Logic
- Examples
 - Process states
 - Queuing system



What can be checked

- Probabilistic Temporal Logics

- CTL can express

- $\langle \rangle (P\{\text{process}=\text{terminate}\} > 0.99)$

- CTL can express

- $\langle \rangle (P\{\text{queue}=\text{full}\} > 0.5)$

- What cannot be expressed

- $\langle \rangle (P\{\text{process}=\text{busy}\} > 1.1 * P\{\text{process}=\text{wait}\})$

- $\langle \rangle (2 < E[\text{queue length}] < 3)$



Model

- Model \square Discrete Time Markov Chain DTMC \square
 - \square With a finite set of states \square s_0, \dots, s_n \square
 - Let \square π \square be a \square conf vector such that \square $\sum_i \pi_i = 1$ \square
 - And with a transition matrix \square M \square such that
 - \square M \square
- Example

```
Markov chain pState          #process states
has states:
    { new, ready, running, waiting, terminating},
transits by :
    [ 0.0, 0.0, 0.0, 0.0, 0.0;
      1.0, 0.7, 0.1, 0.2, 0.0;
      0.0, 0.3, 0.6, 0.0, 0.0;
      0.0, 0.0, 0.1, 0.8, 0.0;
      0.0, 0.0, 0.2, 0.0, 1.0  ]
```



Specification Logic

- Specification logic \square LTL $\square \Psi$
 - The atomic propositions \square of Ψ are inequalities on the \square inf
 - \square ie \square $a_i \square \square \dots \square a_{in} \square \square \square$

- Example

a : $0.999 < P\{pState=terminating\}$,

b : $0.2 < P\{pState=ready\}$,

c : $0.7 < P\{pState=waiting\}$,

d : $P\{pState=waiting\} < 0.2$

$\langle \rangle$ a

$(b \wedge c) \rightarrow \sim \langle \rangle (b \wedge d)$

Specification Logic $\square\text{TL}\square$

- $\square\text{Syntax}\square$

$$\psi ::= T \mid F \mid \text{ineq} \mid$$

$$\neg\psi \mid \psi \vee \phi \mid \psi \wedge \phi \mid$$

$$\mathbf{X}\psi \mid \psi \mathbf{U} \phi \mid \psi \mathbf{R} \phi$$

$$\text{ineq} ::= \sum_{i=1}^n a_i \cdot P\{X = s_i\} < b,$$

- $\square\text{Semantics}\square$

$$\mathbf{M}, \mathbf{x} \models T$$

$$\mathbf{M}, \mathbf{x} \not\models F$$

$$\mathbf{M}, \mathbf{x} \models \sum a_i \cdot P\{X = s_i\} < b \text{ iff } \sum a_i \cdot x_i < b$$

$$\mathbf{M}, \mathbf{x} \models \neg\phi \quad \text{iff } \mathbf{M}, \mathbf{x} \not\models \phi$$

$$\mathbf{M}, \mathbf{x} \models \phi \vee \psi \quad \text{iff } \mathbf{M}, \mathbf{x} \models \phi \text{ or } \mathbf{M}, \mathbf{x} \models \psi$$

$$\mathbf{M}, \mathbf{x} \models \phi \wedge \psi \quad \text{iff } \mathbf{M}, \mathbf{x} \models \phi \text{ and } \mathbf{M}, \mathbf{x} \models \psi$$

$$\mathbf{M}, \mathbf{x} \models \mathbf{X}\phi \quad \text{iff } \mathbf{M}, \mathbf{M} \cdot \mathbf{x} \models \phi$$

$$\mathbf{M}, \mathbf{x} \models \phi \mathbf{U} \psi \quad \text{iff there exists an } i \geq 0 \text{ such that } \mathbf{M}, \mathbf{M}^i \cdot \mathbf{x} \models \psi$$

$$\text{and for all } 0 \leq j < i, \mathbf{M}, \mathbf{M}^j \cdot \mathbf{x} \models \phi$$

$$\mathbf{M}, \mathbf{x} \models \phi \mathbf{R} \psi \quad \text{iff for all } j \geq 0, \text{ if for every } i < j \mathbf{M}, \mathbf{M}^i \cdot \mathbf{x} \not\models \phi$$

$$\text{then } \mathbf{M}, \mathbf{M}^j \cdot \mathbf{x} \models \psi$$

$$X \models \psi \text{ iff for all initial pmf } \mathbf{x}(0), \mathbf{M}, \mathbf{x}(0) \models \psi,$$

Model Checking Tool

<http://osl.bs.cs.uiuc.edu/~ykwon/>

An LTL Model Checker for Discrete Time Markov Chains - Microsoft Internet Explorer

File Edit View Favorites Tools Help

Address <http://osl.cs.uiuc.edu/~ykwon4/>

An LTL Model Checker for D

example:

```
model:
  Markov chain q
  has states :
    { q0, q1, q2, q3, q4, q5},
  transits by :
    [ .94, .07, .00, .00, .00, .00;
      .06, .87, .07, .00, .00, .00;
      .00, .06, .87, .07, .00, .00;
      .00, .00, .06, .87, .07, .00;
      .00, .00, .00, .06, .87, .07;
      .00, .00, .00, .00, .06, .93 ]
specification:
  a : P{q=q4} + 0.7 < P{q=q5},
  b : P{q=q3} + 0.1 < P{q=q4},
  c : P{q=q2} + 0.1 < P{q=q3},
  d : 1+P{q=q1} + 2+P{q=q2} + 3+P{q=q3} + 4+P{q=q4} + 5+P{q=q5} < 2.057,
  e : 1+P{q=q1} + 2+P{q=q2} + 3+P{q=q3} + 4+P{q=q4} + 5+P{q=q5} < 2.056
  a -> <> b
  #a -> <> c
```

Why don't you try it here (copy and paste th

```
model:
  Markov chain q
  has states :
    { q0, q1, q2, q3, q4, q5},
  transits by :
    [ .94, .07, .00, .00, .00, .00;
      .06, .87, .07, .00, .00, .00;
      .00, .06, .87, .07, .00, .00;
      .00, .00, .06, .87, .07, .00;
      .00, .00, .00, .06, .87, .07;
      .00, .00, .00, .00, .06, .93 ]
```

An LTL Model Checker - Microsoft Internet Explorer

File Edit View Favorites Tools Help

Address <http://osl.cs.uiuc.edu/~ykwon4/>

Model and Specification:

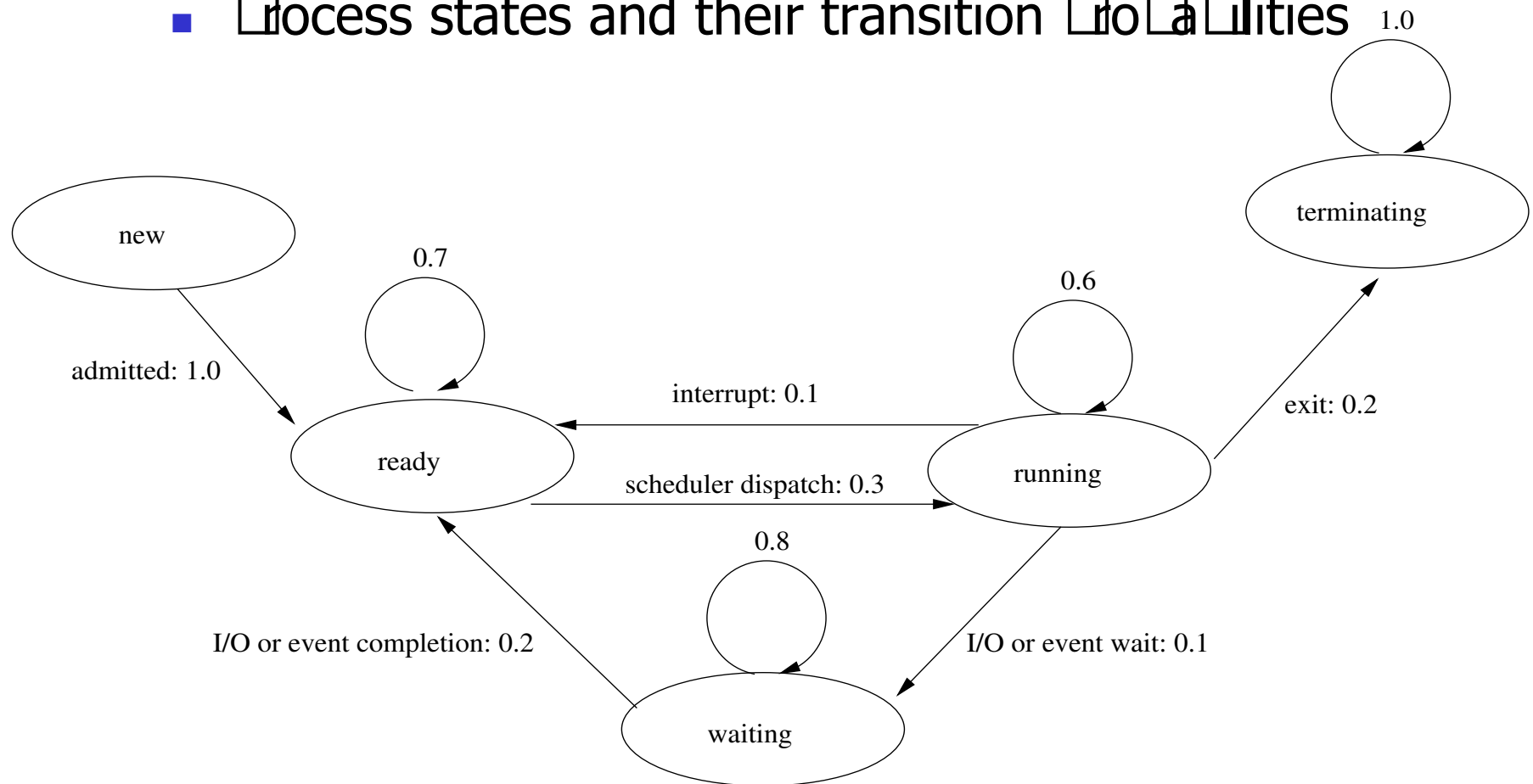
```
model:
  Markov chain q
  has states :
    { q0, q1, q2, q3, q4, q5},
  transits by :
    [ .94, .07, .00, .00, .00, .00;
      .06, .87, .07, .00, .00, .00;
      .00, .06, .87, .07, .00, .00;
      .00, .00, .06, .87, .07, .00;
      .00, .00, .00, .06, .87, .07;
      .00, .00, .00, .00, .06, .93 ]
specification:
  a : P{q=q4} + 0.7 < P{q=q5},
  b : P{q=q3} + 0.1 < P{q=q4},
  c : P{q=q2} + 0.1 < P{q=q3},
  d : 1+P{q=q1} + 2+P{q=q2} + 3+P{q=q3} + 4+P{q=q4} + 5+P{q=q5} < 2.057,
  e : 1+P{q=q1} + 2+P{q=q2} + 3+P{q=q3} + 4+P{q=q4} + 5+P{q=q5} < 2.056
  a -> <> b
  #a -> <> c
```

Result

Depth: 116
Result: T

Operating System Process States

- Process states and their transition probabilities





Example process states

model:

```
Markov chain pState          #process states
has states:
  { new, ready, running, waiting, terminating},
transits by :
  [ 0.0, 0.0, 0.0, 0.0, 0.0;
    1.0, 0.7, 0.1, 0.2, 0.0;
    0.0, 0.3, 0.6, 0.0, 0.0;
    0.0, 0.0, 0.1, 0.8, 0.0;
    0.0, 0.0, 0.2, 0.0, 1.0  ]
```

specification:

```
a : 0.999 < P{pState=terminating},
b : 0.2 < P{pState=ready},
c : 0.7 < P{pState=waiting},
d : P{pState=waiting} < 0.2
```

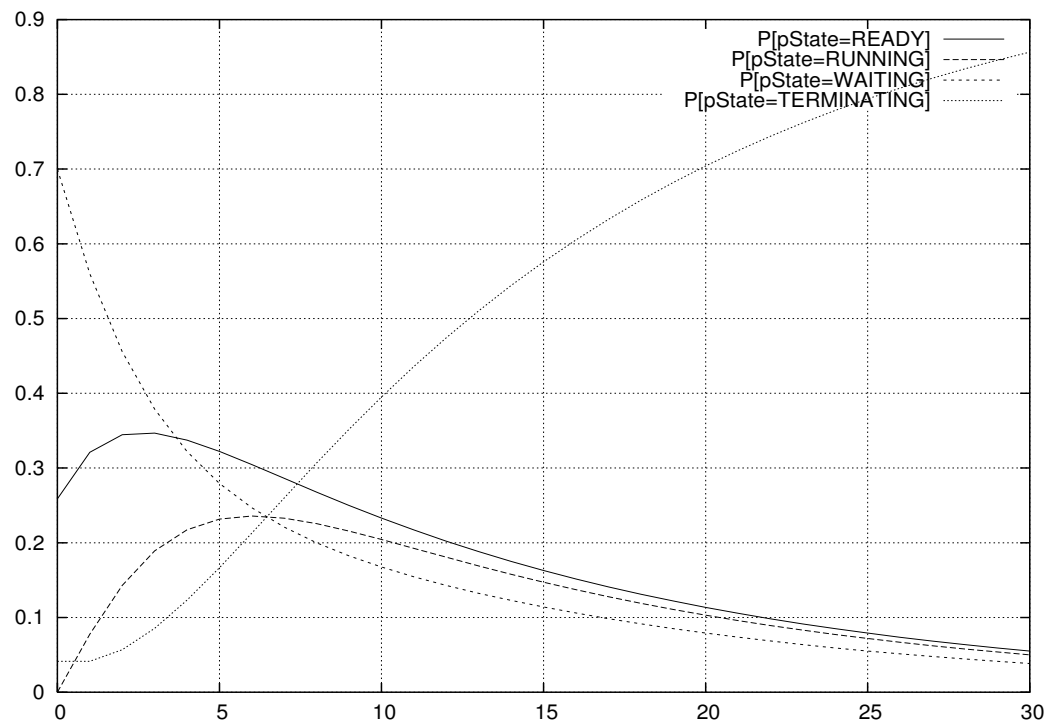
```
#<> a
```

```
(b /\ c) -> ~ <> (b /\ d)
```

Example process states

```
<>a  
Depth: 116  
Result: T
```

```
(b /\ c) -> ~ <> (b /\ d)  
Depth: 116  
Result: F  
counter example:  
[ 0.00 0.26 0.00 0.70 0.04 ]
```



ERROR: invalidrestore
OFFENDING COMMAND: restore

STACK:

-savelevel-
-savelevel-
-dictionary-